CHAPTER 9

FARADAY’S LAW AND DISPLACEMENT CURRENT

9.1 FARADAY’S LAW

9.1.1 TIME VARYING FIELD – STATIONARY CIRCUIT

9.1.2 MOVING CIRCUIT – STATIC FIELD

9.1.3 TIME VARYING FIELD - MOVING CIRCUIT

9.2 DISPLACEMENT CURRENT

9.3 LOSSY DIELECTRICS

9.4 BOUNDARY CONDITIONS

9.0 FARADAY'S LAW AND DISPLACEMENT CURRENT

Two topics will be discussed :

(i) Faraday's Law – about the existence of electromotive force (emf) in the magnetic field

(ii) Displacement current – that exists due to time varying field

That will cause the modification of Maxwell's equations (in point form - static case) studied previously and hence becomes a concept basic to the understanding of all fields in electrical engineering.

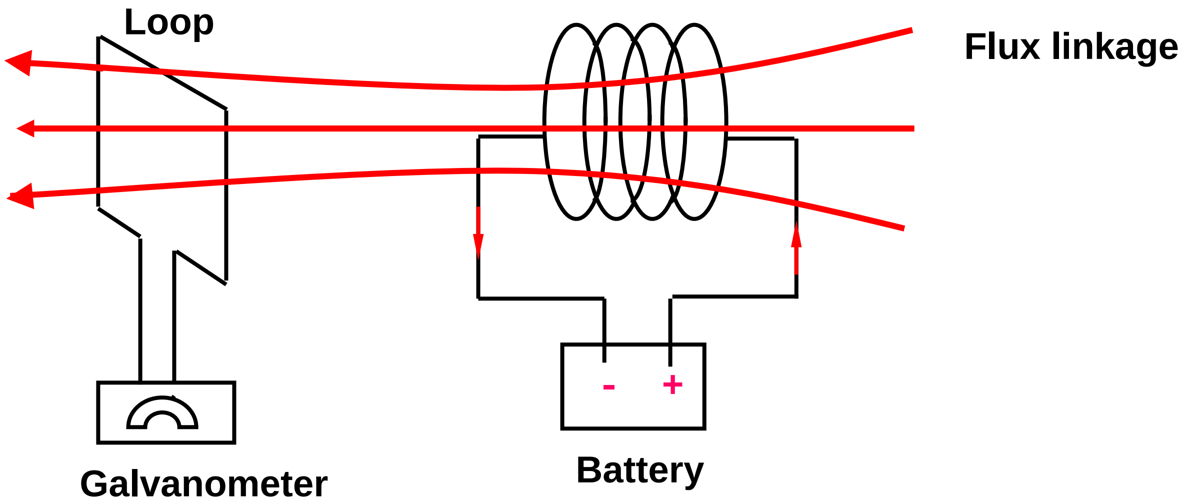
9.1 FARADAY'S LAW

Michael Faraday ***– proved that if the current can produce magnetic field, the reverse also will be true.***

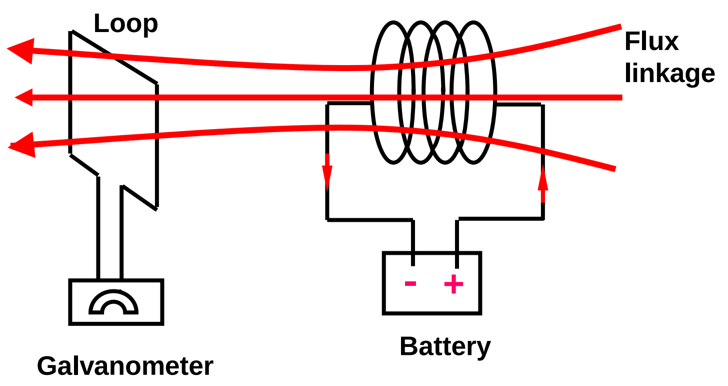
Proven only after 10 years in 1831.

The magnetic field can produce current in a loop, only if the magnetic flux linkage the surface of the loop is time varying.

Faraday's Experiment :



- ***Current produced magnetic field and the magnetic flux is given by :***



$$\psi_m = \int_s \overline{B} \bullet d\overline{s} \qquad (1)$$

- ***No movement in galvanometer means that the flux is constant.***
- ***Once the battery is put off – there is a movement in the galvanometer needle.***
- ***The same thing will happen once the battery is put on - but this time the movement of the needle is in the opposite direction.***

Conclusions : The current was induced in the loop

- ***when the flux varies***

- ***once the battery is connected***

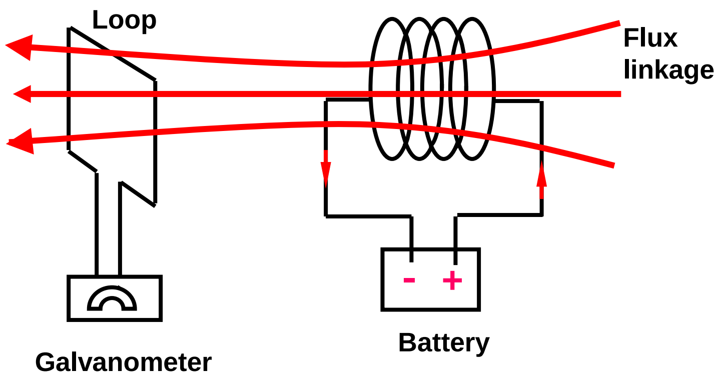
- ***if the loop is moving or rotating***

Induced current will induced electromotive voltage or induced emf V_{emf} given by :

$$V_{emf} = -N\frac{\partial \psi}{\partial t} = -N\frac{\partial}{\partial t}\int_s \overline{B}\cdot d\overline{s} \quad (V) \qquad (2)$$

where   N = number of turns

Equation (2) is called Faraday's Law



Lenz's Law summarizes the –ve sign is that : The induced voltage established opposes the the flux produced by the loop.

In general, Faraday's law manifests that the V_{emf} can be established in these 3 conditions :

- ***Time varying field – stationary circuit (Transformer emf)***
- ***Moving circuit – static field (Motional emf)***
- ***Time varying field - Moving circuit (both transformer emf and motional emf exist)***

9.1.1 TIME VARYING FIELD – STATIONARY CIRCUIT (TRANSFORMER EMF)

$$V_{emf} = -N\int_s \frac{\partial \overline{B}}{\partial t}\cdot d\overline{s} \quad (V) \qquad (3)$$

V_{emf} = the potential difference at terminal 1 and 2.

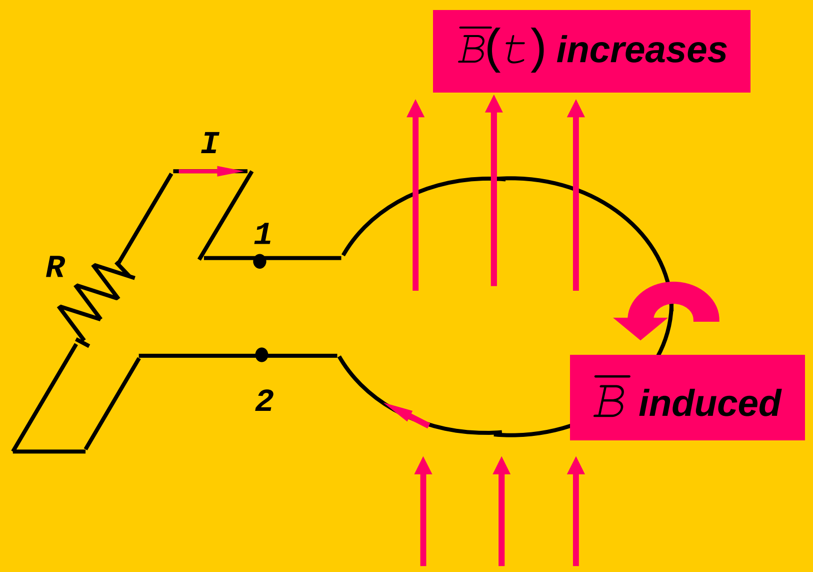


Diagram shows a circle loop with the surface area S placed in the magnetic field $\overline{B}(t)$

From electric field :

$$V_{emf} = \oint_\ell \overline{E}\bullet d\overline{\ell} \qquad (4)$$

If N = 1 :

$$V_{emf} = \oint_\ell \overline{E}\bullet d\overline{\ell} = -\int_s \frac{\partial \overline{B}}{\partial t}\bullet d\overline{s} \qquad (5)$$

$$V_{emf} = \oint_{\ell} \overline{E} \bullet d\overline{\ell} = -\int_{s} \frac{\partial \overline{B}}{\partial t} \bullet d\overline{s}$$

Using Stoke's theorem :

$$\int_{s} \left(\nabla \times \overline{E} \right) \bullet d\overline{s} = -\int_{s} \frac{\partial \overline{B}}{\partial t} \bullet d\overline{s} \qquad (6)$$

Hence Maxwell's equation becomes :

$$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t} \qquad (7)$$

9.1.2 MOVING CIRCUIT – STATIC FIELD (MOTIONAL EMF)

Force :

$$\overline{F}_m = q\left(\overline{u} \times \overline{B}\right)$$

$$\overline{E}_m = \frac{\overline{F}_m}{q} = \left(\overline{u} \times \overline{B}\right)$$

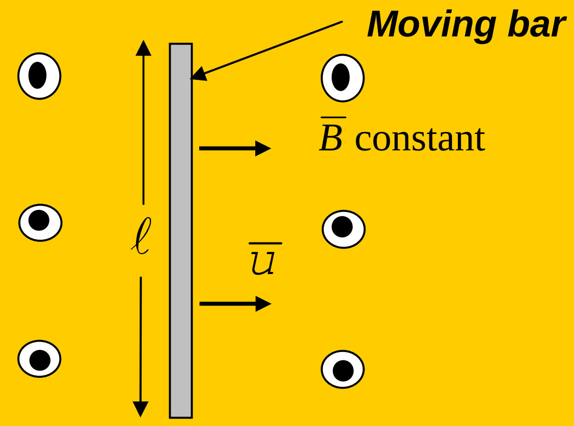


Diagram shows a bar moving with a velocity $\overline{u}$ in a static field $\overline{B}$.

Hence :

$$V_{emf} = \oint \overline{E}_m \bullet d\overline{\ell} = \oint \left(\overline{u} \times \overline{B}\right) \bullet d\overline{\ell}$$

Fleming's Right hand rule

Thumb – Motion

1st finger – Field

Second finger - Current

9.1.3 TIME VARYING FIELD - MOVING CIRCUIT

Both transformer emf and motional emf exist

$$V_{emf} = \oint \overline{E} \bullet d\overline{\ell} = \int_{s} -\frac{\partial \overline{B}}{\partial t} \bullet d\overline{s} + \oint \left(\overline{u} \times \overline{B} \right) \bullet d\overline{\ell}$$

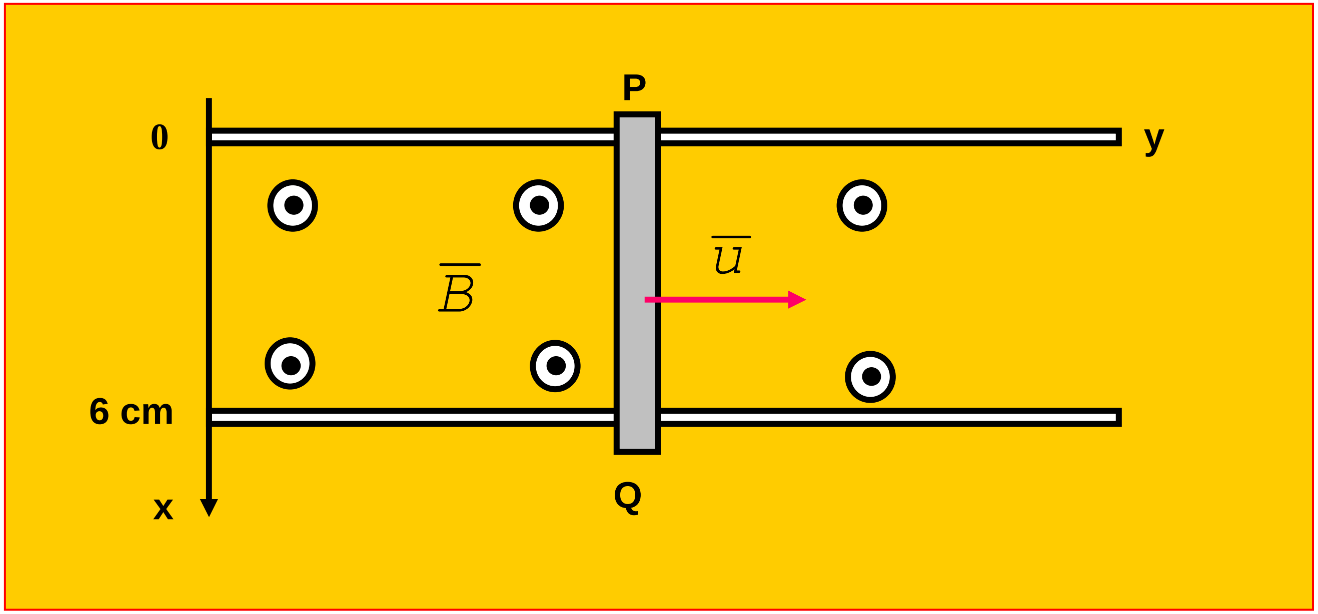
Ex. 9.1 : ***A coducting bar moving on the rail is shown in the diagram. Find an induced voltage on the bar if :***

(i) A bar position at y = 8 cm and $\overline{B} = 4\cos 10^6 t\, \hat{z}\ mWb/m^2$

(ii) A bar moving with a velocity $\overline{u} = 20\hat{y}\ m/s$ ***and*** $\overline{B} = 4\hat{z}\ mWb/m^2$

(iii) A bar moving with a velocity $\overline{u} = 20\hat{y}\ m/s$ ***and***

$\overline{B} = 4\cos(10^6 t - y)\hat{z}\ mWb/m^2$



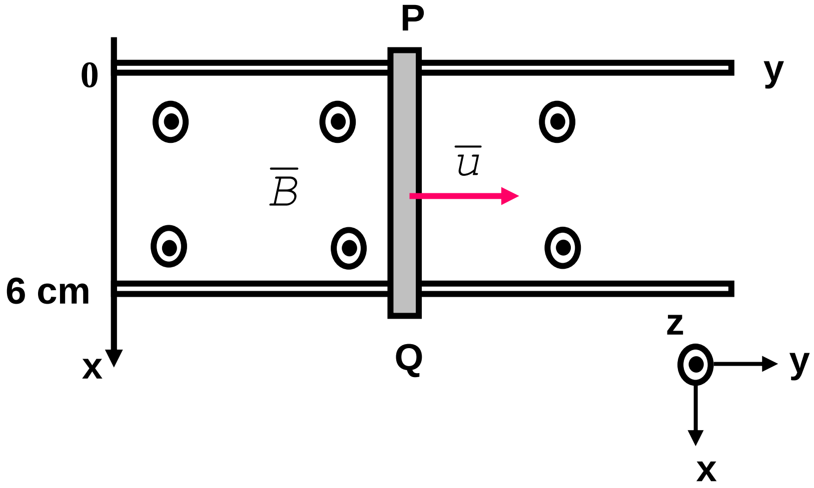
Solution :

(i) Transformer case :

$$V_{emf} = -\int \frac{\partial \overline{B}}{\partial t} \bullet d\overline{s}$$

$$= \int_{y=0}^{0.08} \int_{x=0}^{0.06} 4(10^{-3})(10^{6}) \sin 10^{6} t \, dxdy$$

$$= 19.2 \sin 10^{6} t \ (V)$$



$$\overline{B} = 4\cos 10^{6} t \ \hat{z} \ mWb/m^{2}$$

According to Lenz's law when $\overline{B}$ increases point P will be at the higher potential with respect to point Q. (B induced will oppose the increasing $\overline{B}$)

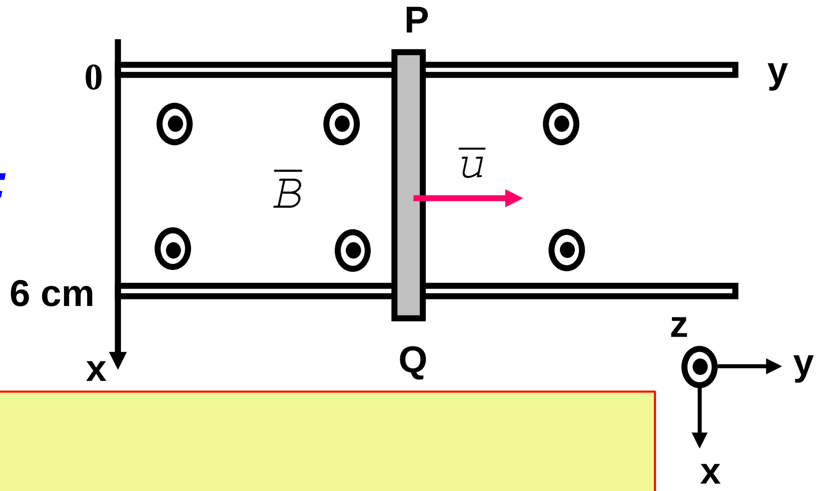
(ii) Motional case :

$$V_{emf} = \int (\overline{u} \times \overline{B}) \bullet d\overline{\ell}$$

$$= \int_{x=0.06}^{0} (20\,\hat{y} \times 4\,\hat{z}) \bullet dx \ \hat{x}$$

$$= -4.8 \ mV$$

Remember : the direction of $d\overline{\ell}$ is opposed the current induced in the loop.



(iii) Both transformer and motional case :

$$\overline{B} = 4\cos(10^6 t - y)\,\hat{z}\ mWb/m^2$$

$$V_{emf} = -\int \frac{\partial \overline{B}}{\partial t} \bullet d\overline{s} + \int (\overline{u} \times \overline{B}) \bullet d\overline{\ell}$$

$$= \int_{x=0}^{0.06} \int_{0}^{y} 4(10^{-3})(10^{6}) \sin(10^{6} t - y')\, dy'\, dx$$

$$+ \int_{0.06}^{0} \left[20\,\hat{y} \times 4(10^{-3}) \cos(10^{6} t - y)\,\hat{z} \right] \bullet dx\,\hat{x}$$

$$= 240\cos(10^{6} t - y')\Big|_{0}^{y} - 80(10^{-3})(0.06)\cos(10^{6} t - y)$$

$$= 240\cos(10^{6} t - y) - 240\cos 10^{6} t - 4.8(10^{-3})\cos(10^{6} t - y)$$

$$\approx 240\cos(10^{6} t - y) - 240\cos 10^{6} t$$

$$V_{emf} = -\int \frac{\partial \overline{B}}{\partial t} \bullet d\overline{s} + \int (\overline{u} \times \overline{B}) \bullet d\overline{\ell}$$

$$\approx 240\cos(10^6 t - y) - 240\cos 10^6 t$$

From trigonometry :

$$\cos A - \cos B = 2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

$$V_{emf} = 480\sin\left(\left(10^6 t - \frac{y}{2}\right)\right)\sin(-\frac{y}{2})$$

9.2 DISPLACEMENT CURRENT

From continuity of current equation :

$$\nabla \bullet \overline{J} = -\frac{\partial \rho_v}{\partial t} \quad A/m^3$$

and

$$\overline{J} = \nabla \times \overline{H}$$

$$\nabla \bullet \nabla \times \overline{H} = -\frac{\partial \rho_v}{\partial t} = -\frac{\partial}{\partial t}\nabla \bullet \overline{D} = -\nabla \bullet \frac{\partial \overline{D}}{\partial t} \quad A/m^3$$

$$\nabla \bullet \nabla \times \overline{H} = 0 = -\frac{\partial \rho_v}{\partial t} + \frac{\partial \rho_v}{\partial t} \quad ; \quad \textbf{and} \quad \nabla \bullet \overline{D} = \rho_v$$

Hence :

$$\nabla \bullet \nabla \times \overline{H} = \nabla \bullet \overline{J} + \frac{\partial}{\partial t}\nabla \bullet \overline{D}$$

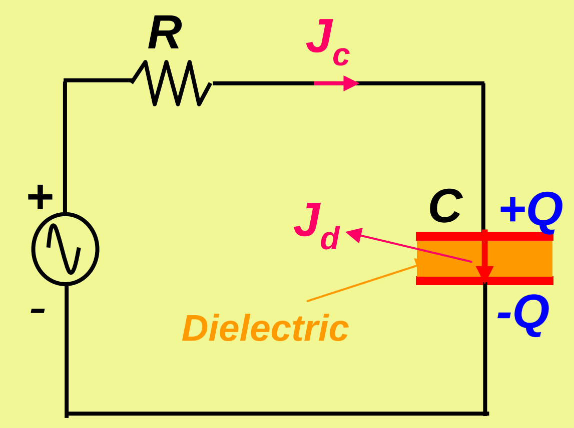
$$\nabla \bullet \nabla \times \overline{H} = \nabla \bullet \overline{J} + \frac{\partial}{\partial t} \nabla \bullet \overline{D}$$

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$$

where :

$\overline{J}$ = ***Conduction current density***

$\frac{\partial \overline{D}}{\partial t}$ = ***Displacement current density***



Hence :

$$\nabla \times \overline{H} = \overline{J}_c + \frac{\partial \overline{D}}{\partial t} = \overline{J}_c + \overline{J}_d$$

Therefore from Faraday's law and the concept of displacement current we can conclude that both the magnetic and electric fields are interrelated.

Maxwell's Equations

Differential Form	Integral Form	Label
$\nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$	$\oint \overline{E} \bullet d\overline{\ell} = -\int \frac{\partial \overline{B}}{\partial t} \bullet d\overline{s}$	Faraday's Law
$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t}$	$\oint \overline{H} \bullet d\overline{\ell} = I + \int \frac{\partial \overline{D}}{\partial t} \bullet d\overline{s}$	Ampere's Circuital Law
$\nabla \bullet \overline{D} = \rho_v$	$\oint_s \overline{D} \bullet d\overline{s} = \int_v \rho_v dv$	Gauss'S Law for Electric Field
$\nabla \bullet \overline{B} = 0$	$\oint_s \overline{B} \bullet d\overline{s} = 0$	Gauss's Law for Magnetic Field

An integral form of Maxwell's equation can be found either by using Divergence Theorem or Stoke Theorem. All electromagnetic (EM) waves must conform or obey all the four Maxwell's equations.

Ex.9.2: ***A parallel plate capacitor having a plate area of 5 cm² and where the plates are separated by a distance of 3 mm is connected to a supply voltage, 50 sin 10³ t Volt. Calculate the displacement current if the dielectric permittivity between the plate is*** $\varepsilon = 2\varepsilon_0$.

Solution :

$$\overline{D} = \varepsilon\overline{E} = \varepsilon V / d$$

$$\overline{J}_d = \frac{\partial \overline{D}}{\partial t} = \frac{\varepsilon}{d}\frac{\partial V}{\partial t}$$

$$\therefore I_d = \overline{J}_d \bullet d\overline{s}$$

$$= \frac{\varepsilon S}{d}\frac{\partial V}{\partial t}$$

$$= 2\varepsilon_0 \frac{5\times 10^{-4}}{3\times 10^{-3}}(10^3)50 \cos 10^3 t$$

$$= 147.4 \cos 10^3 t \ nA$$

This example is to show the use of Maxwell's equation and the inter relation of electric field and magnetic field.

Ex.9.3: Given a magnetic medium with characteristics $\sigma = 0, \mu = 2\mu_0, \varepsilon = 5\varepsilon_0$ **has** $\overline{H} = 2\cos(\omega t - 3y)\hat{z}\ A/m$ **. Find** ω **and** $\overline{E}$ **.**

Solution :

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t} \quad ; \quad \sigma = 0$$

$$\nabla \times \overline{H} = \varepsilon \frac{\partial \overline{E}}{\partial t} \rightarrow \overline{E} = \frac{1}{\varepsilon}\int (\nabla \times \overline{H})dt$$

$$\nabla \times \overline{H} = \begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & H_z \end{vmatrix} = \frac{\partial H_z}{\partial y}\hat{x} - \frac{\partial H_z}{\partial x}\hat{y}$$

$$= 6\sin(\omega t - 3y)\hat{x}$$

$$\therefore \overline{E} = \frac{1}{\varepsilon}\int 6\sin(\omega t - 3y)dt\ \hat{x}$$

$$= -\frac{1}{5\omega\varepsilon_0} 6\cos(\omega t - 3y)\ \hat{x}$$

$$\nabla \times \overline{E} = -\mu \frac{\partial \overline{H}}{\partial t}, \rightarrow \overline{H} = -\frac{1}{\mu}\int (\nabla \times \overline{E})\, dt$$

$$\nabla \times \overline{E} = \begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ E_x & 0 & 0 \end{vmatrix} = -\frac{\partial E_x}{\partial z}\hat{y} + \frac{\partial E_x}{\partial y}\hat{z}$$

$$= \frac{-3(6)}{5\omega\varepsilon_0}\sin(\omega t - 3y)\,\hat{z}$$

$$\overline{H} = \frac{1}{\mu}\frac{18}{5\omega\varepsilon_0}\int \sin(\omega t - 3y)\, dt\ \hat{z}$$

$$= \frac{18}{10\omega\mu_0\varepsilon_0}\cos(\omega t - 3y)\,\hat{z}$$

$$\therefore\ \overline{E} = \frac{1}{\varepsilon}\int 6\sin(\omega t - 3y)dt\ \hat{x}$$

$$= -\frac{1}{5\omega\varepsilon_0} 6\cos(\omega t - 3y)\,\hat{x}$$

Compare :

$$\overline{H} = 2\cos(\omega t - 3y)\hat{z}\ A/m$$

$$\frac{18}{10\omega\mu_0\varepsilon_0} = 2$$

$$\rightarrow \omega = 2.846\times 10^8\ rad/s$$

Hence : $$\overline{E} = -476.8\cos(2.846\times 10^8 t - 3y)\,\hat{x}\ \ (V/m)$$

9.3 LOSSY DIELECTRICS

Main function of dielectric material is to be used as an insulator.

For a perfect dielectric : $\sigma = 0$

Hence Maxwell's equation :

$$\nabla \times \overline{H} = (\sigma + j\omega\varepsilon')\overline{E}$$

$$\nabla \times \overline{H} = j\omega\varepsilon\overline{E} \quad \textbf{(1)}$$

For lossy dielectric : $\sigma \neq 0$

$$\nabla \times \overline{H} = (\sigma + j\omega\varepsilon')\overline{E} \quad \textbf{(2)}$$

Compare (1) and (2) :

$$\sigma + j\omega\varepsilon' = j\omega\varepsilon$$

$$\varepsilon = \varepsilon' - j\frac{\sigma}{\omega}$$

$$\varepsilon = \varepsilon'\left(1 - j\frac{\sigma}{\omega\varepsilon'}\right) = \varepsilon' - j\varepsilon''$$

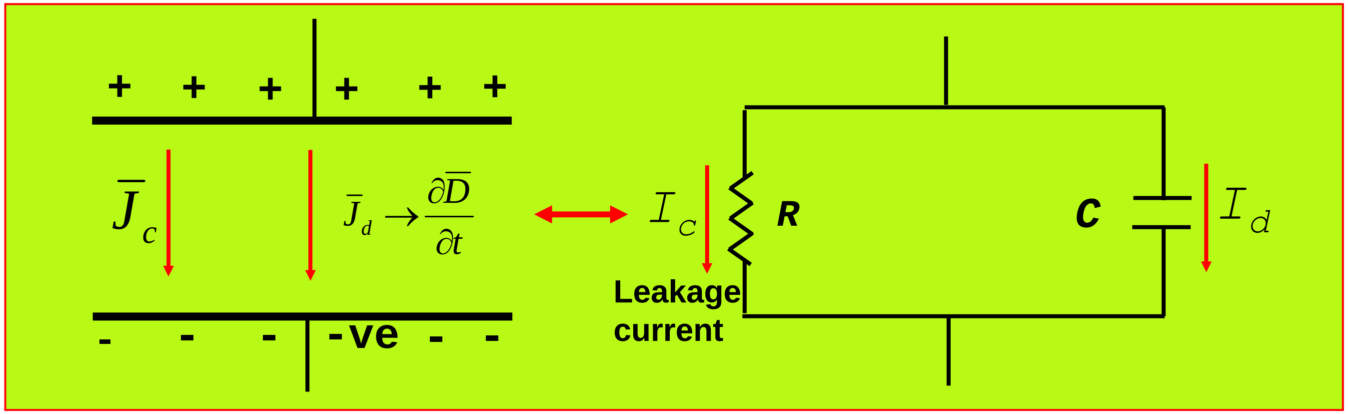
where :

$\sigma / \omega\varepsilon'$ **= loss tangent**

Loss tangent is the ratio of the magnitude of the conduction current density to the magnitude of the displacement current density $= \dfrac{|I_c|}{|I_d|}$

A lossless capacitor has a loss tangent of zero.

For lossy capacitor, an equivalent circuit can be replaced by its equivalent resistance in parallel with a perfect capacitor as shown in the diagram :



$$I_c = GV = \frac{V}{R} \quad \textbf{and} \quad I_d = j\omega CV$$

From page 110 & 111

$$R = \frac{V}{I} = \frac{\int \bar{E} \cdot d\bar{l}}{\int \bar{J} \cdot d\bar{s}}$$

$$C = \frac{Q}{V} = \frac{\int \bar{D} \cdot d\bar{s}}{\int \bar{E} \cdot d\bar{l}}$$

Hence loss tangent :

$$\frac{|I_c|}{|I_d|} = \frac{1/R}{\omega C} = \frac{\sigma s/d}{\omega \varepsilon' s/d} = \frac{\sigma}{\omega \varepsilon'} = \frac{\varepsilon''}{\varepsilon'}$$

Ex.9.4: ***Find the average power loss per unit volume for a capacitor having the following properties; dielectric constant 2.5 loss tangent 0.0005 for an applied electric field intensity of 1 kV/m at frequency 500 MHz.***

Solution :

$$\text{Loss tangent} = 0.0005 = \frac{\sigma}{\omega\varepsilon'}$$

$$\rightarrow \sigma = (0.0005)(2\pi)(500\times10^{6})(2.5\varepsilon_0)$$

$$= 3.476\times10^{-5}\ S/m$$

$$P = \frac{V^2}{2R} = \frac{V^2}{2\,{}^{d}\!/_{\sigma s}} = {}^{1}\!/_{2}\,E^2\sigma s d$$

$$\therefore \frac{P}{volume} = {}^{1}\!/_{2}\,\sigma E^2 = {}^{1}\!/_{2}(3.476\times10^{-5})(10^{3})^{2}$$

$$= 17.38\ (W/m^3)$$

9.4 BOUNDARY CONDITIONS

Boundary conditions for time varying field are the same as boundary conditions in electrostatics and magnetostatics fields.

Tangential components for $\bar{E}$ and $\bar{H}$:

$$E_{1t} = E_{2t} \quad ; \quad H_{1t} - H_{2t} = J_s$$

Normal components :

$$D_{1n} - D_{2n} = \rho_s \quad ; \quad B_{1n} = B_{2n}$$

If medium 2 is perfect conductor :

$$\bar{E} = \bar{H} = 0$$

Hence :

$$E_{1t} = 0 \; ; \; H_{1t} = J_s$$

$$D_{1n} = \rho_s \; ; \; B_{1n} = 0$$