FIELDS AND MATERIALS

INTRODUCTION

- In our discussion of electric field thus far, we considered the medium to be free space and source to be static charges.
- We shall now introduce materials.

Contain charged particles that respond to applied field

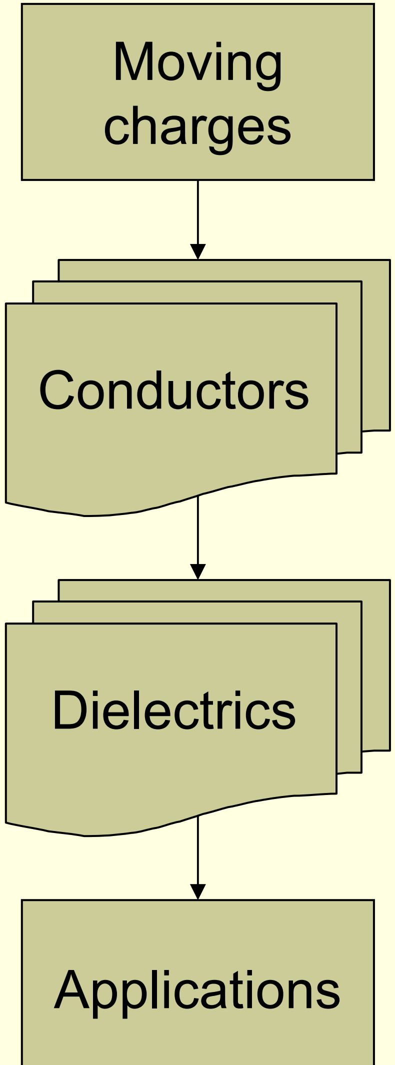
Depending on their response, they may be classified as

Conductor

Semiconductor

Dielectric

APPROACH



Study moving charges in free space.

Conductivity (σ)
Ohm's law
Resistance from field theory
Conductor properties under static conditions
Conductor – Free space boundary conditions

Dielectric properties under static conditions
Dielectric - dielectric boundary conditions

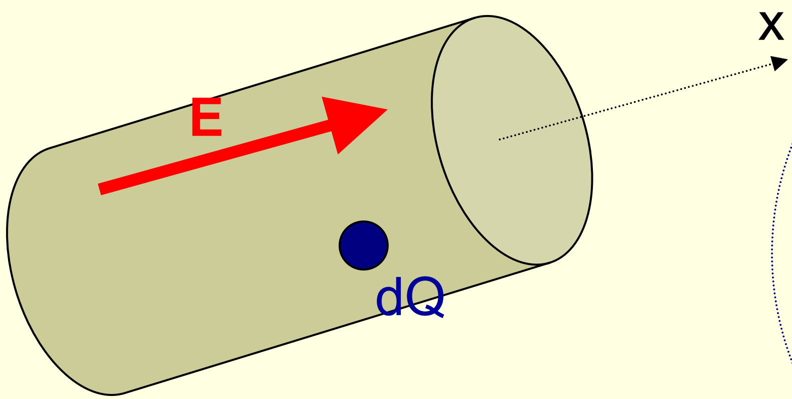
Capacitor

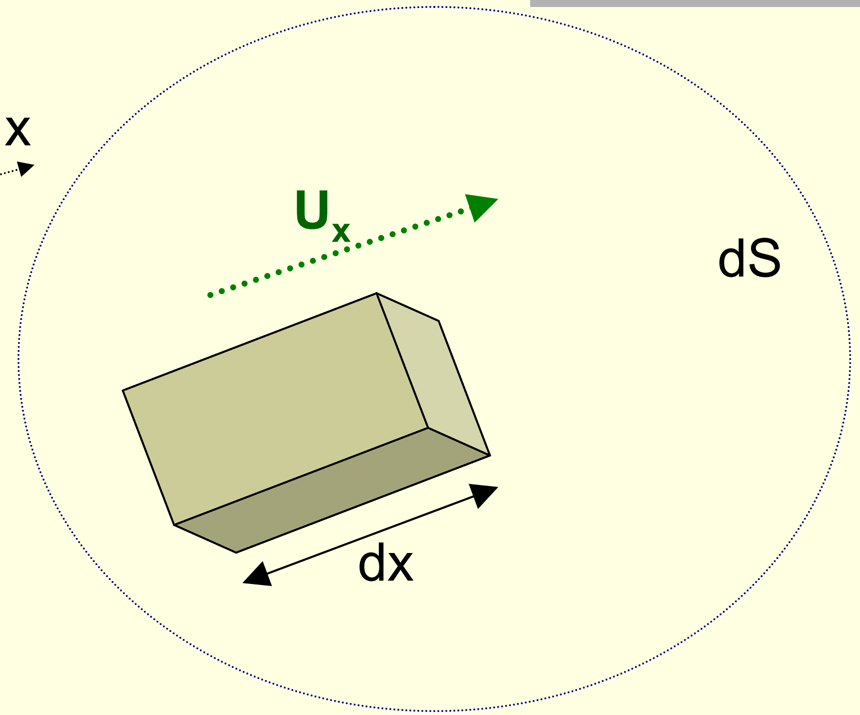
MOVING CHARGES

- OBJECTIVES
  - To introduce concept of current in free space.
  - To define convection current and convection current density.
  - To explain in your own words the concept of continuity current equation.

DEFINITION

Long cylinder of ρ_v





Exploded view of sample dQ

Current is defined as the movement of charge through a given surface and is equal to the coulombs per second through that surface.

$$dI = \lim_{dt \to 0}\left[\frac{dQ}{dt}\right]$$

MATHEMATICAL MODEL

$$dI = \lim_{dt \to 0}\left[\rho_v \frac{dx}{dt} dS\right] = \rho_v U_x dS$$ → Convection current

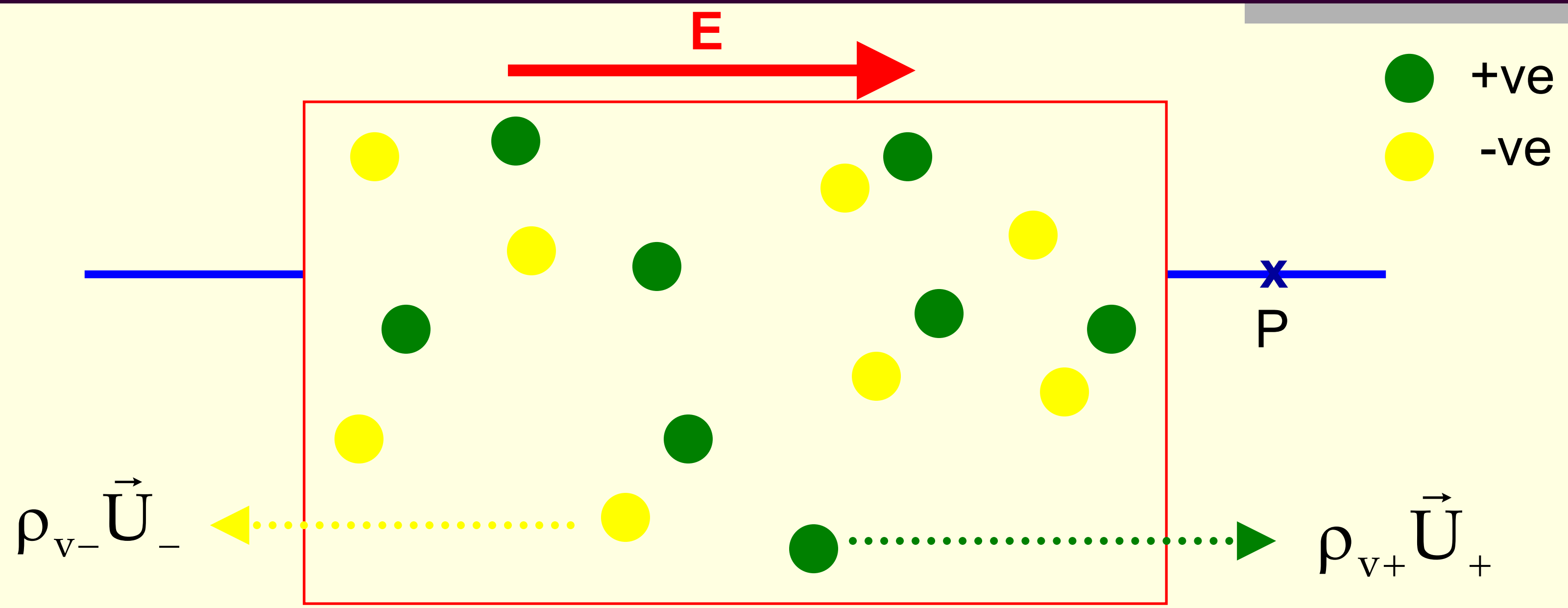
Divide by dS, introducing current density

$$J_x = \lim_{dS \to 0}\left[\frac{dI}{dS}\right] = \rho_v U_x$$

Written in general vector form

$$\vec{J} = \rho_v \vec{U}$$ → Convection current density

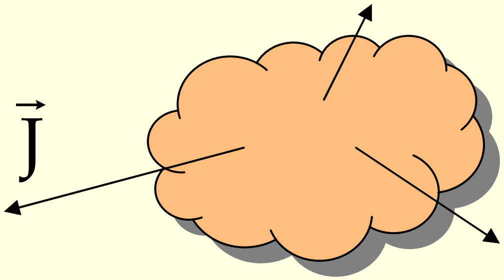
Consider a material where both positive and negative charges exist



At point P, current density ⇒ $\vec{J} = \rho_{v+}\vec{U}_+ + \rho_{v-}\vec{U}_-$

Total current ⇒ $I = \int_S \vec{J} \cdot d\vec{S}$

CONTINUITY OF CURRENT EQUATION



$$I = \oint_S \vec{J} \cdot d\vec{S} = -\frac{\partial Q_{en}}{\partial t}$$

Divergence theorem

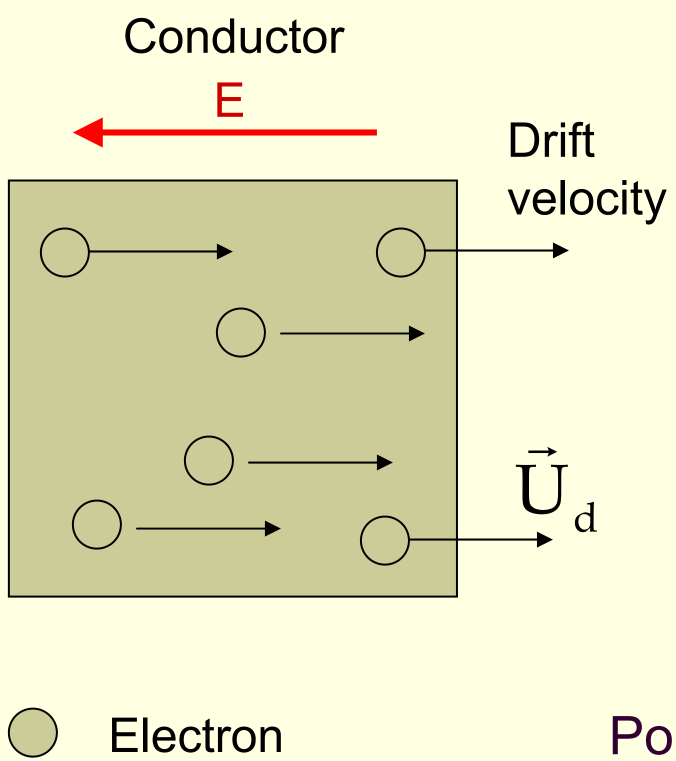
$$Q_{en} = \int_v \rho_v dv$$

$$\int_v \nabla \cdot \vec{J} dv \qquad - \int_v \frac{\partial \rho_v}{\partial t} dv$$

The point form of the continuity of the current equation

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \quad (\mathrm{A\,m^{-3}})$$

CONDUCTIVITY



$$\vec{U}_d = -\mu_e \vec{E}$$

μ_e: Electron mobility

$$\vec{J} = -\rho_{ve}\mu_e \vec{E}$$

$-\rho_{ve}\mu_e$: Conductivity

Point form of Ohm's law

$$\vec{J} = \sigma \vec{E}$$

CONDUCTIVITY OF SEVERAL MATERIALS

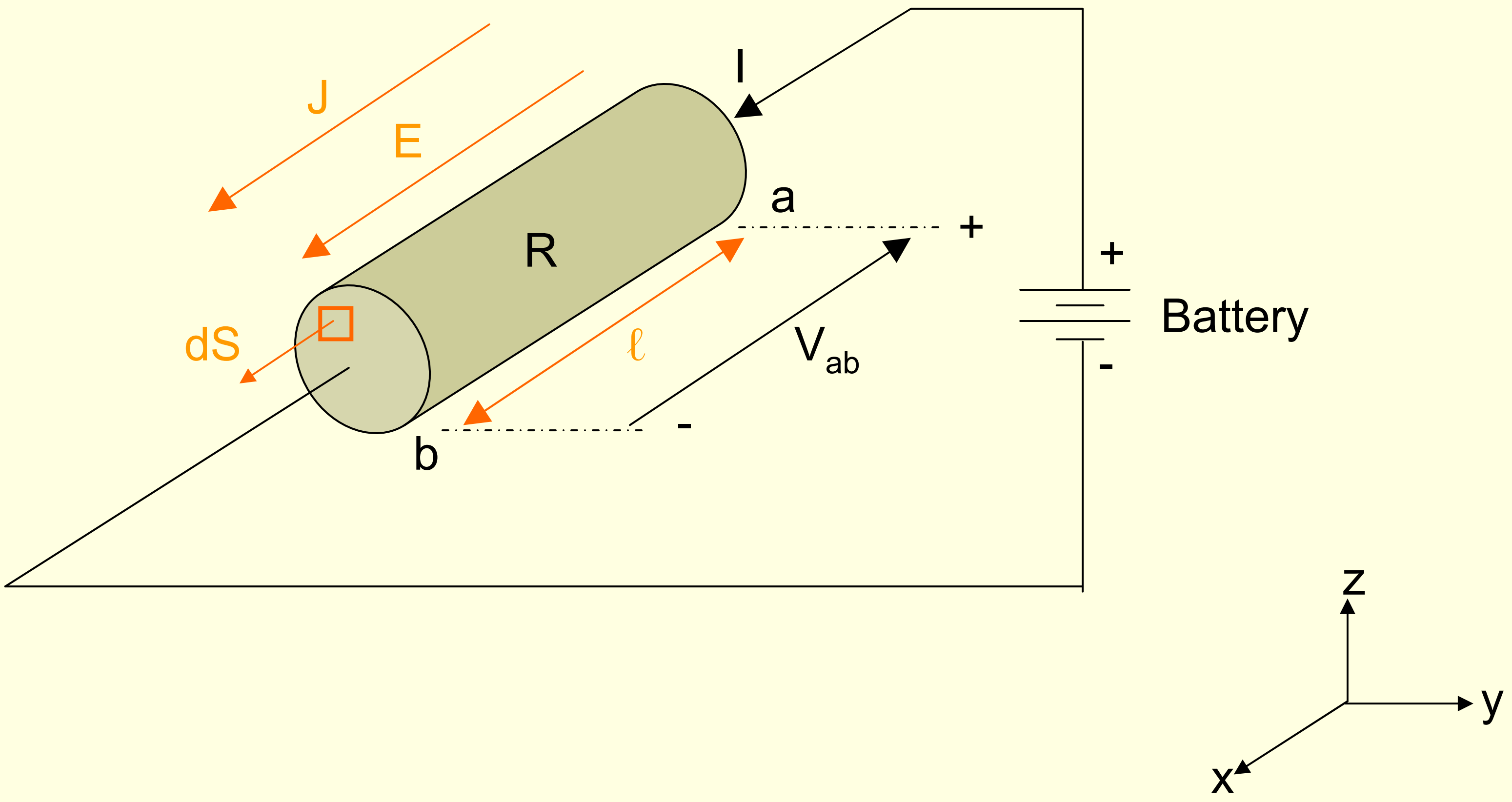
Material	σ (S m^{-1})
Silver	6.17×10^{7}
Iron	1.03×10^{7}
Ferrite	1.0×10^{-2}
Intrinsic silicon	0.44×10^{-3}
Distilled water	1.0×10^{-4}
Mica	1.0×10^{-15}

Conductors

Intrinsic semiconductors

Insulators

RESISTANCE FROM FIELD THEORY



MATHEMATICAL EQUATION

$$R = \frac{V_{ab}}{I} = \frac{-\int_b^a \left(\vec{E}\cdot d\vec{\ell}\right)}{\int_s \sigma\vec{E}\cdot d\vec{S}} = \frac{\int_b^a \left(-\frac{\vec{J}}{\sigma}\cdot d\vec{\ell}\right)}{\int_s \vec{J}\cdot d\vec{S}}$$

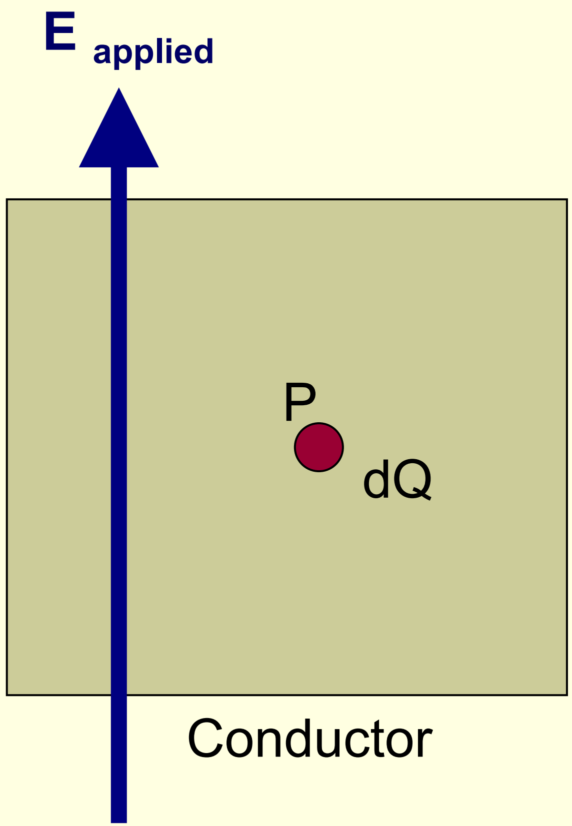
Electrical circuit analysis

From field theory

This concept is useful to conducting material of non-uniform cross section and non-uniform E or J.

CONDUCTOR PROPERTIES UNDER STATIC CONDITIONS

MODEL



Point form of the continuity of current equation

$$\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \quad (\mathrm{A\,m^{-3}})$$

Point form of Ohm's law

$$\vec{J} = \sigma \vec{E}$$

$$\nabla \cdot \vec{E} = -\frac{1}{\sigma}\left(\frac{\partial \rho_v}{\partial t}\right)$$

Point form of Gauss's law

$$\nabla \cdot \vec{E} = \rho_v / \varepsilon_0$$

Simple partial differential equation

$$\frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\varepsilon_0}\rho_v = 0$$

SOLUTION TO PARTIAL DIFFERENTIAL EQUATION

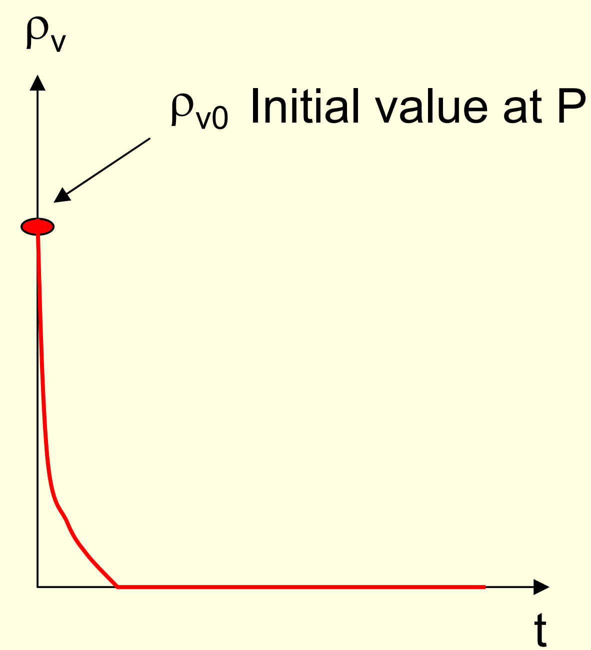
At point P

$$\rho_V = \rho_V(t) = \rho_{v0} e^{-(\sigma/\varepsilon_0)t} = \rho_{v0} e^{-t/t_r}$$

Charge density at t = 0 (ρ_{v0})

Relaxation time constant

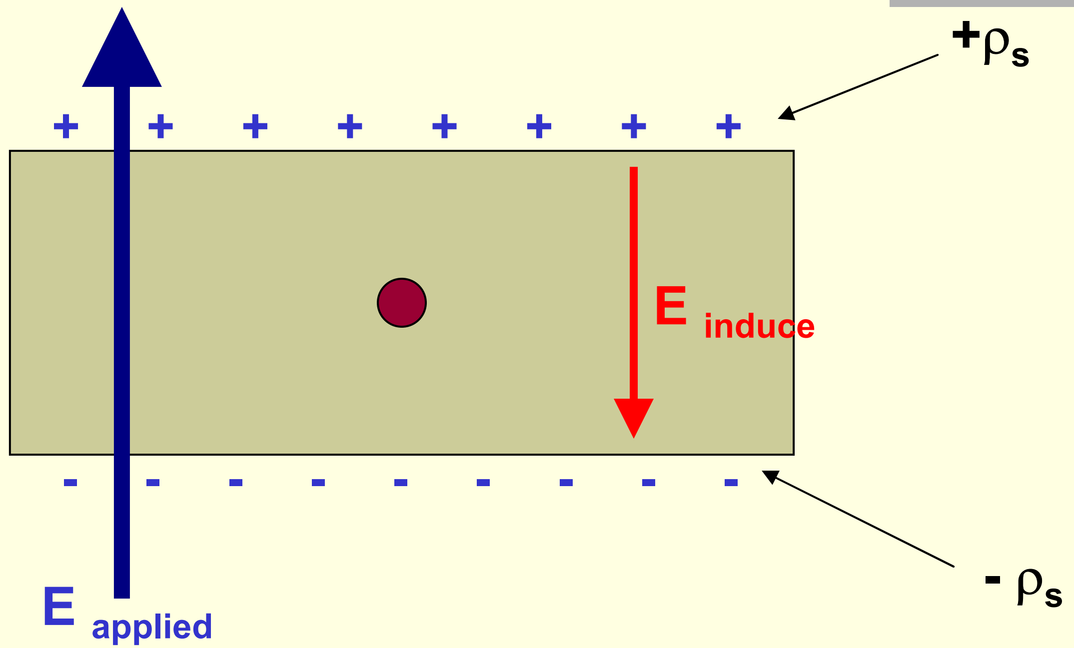
$$t_r = \frac{\varepsilon_0}{\sigma}$$



For conductor

$$\sigma \to \infty \Rightarrow t_r \to \text{small}$$

ANALYSIS



$$E_{net} = E_{applied} - E_{induce} \Rightarrow 0$$

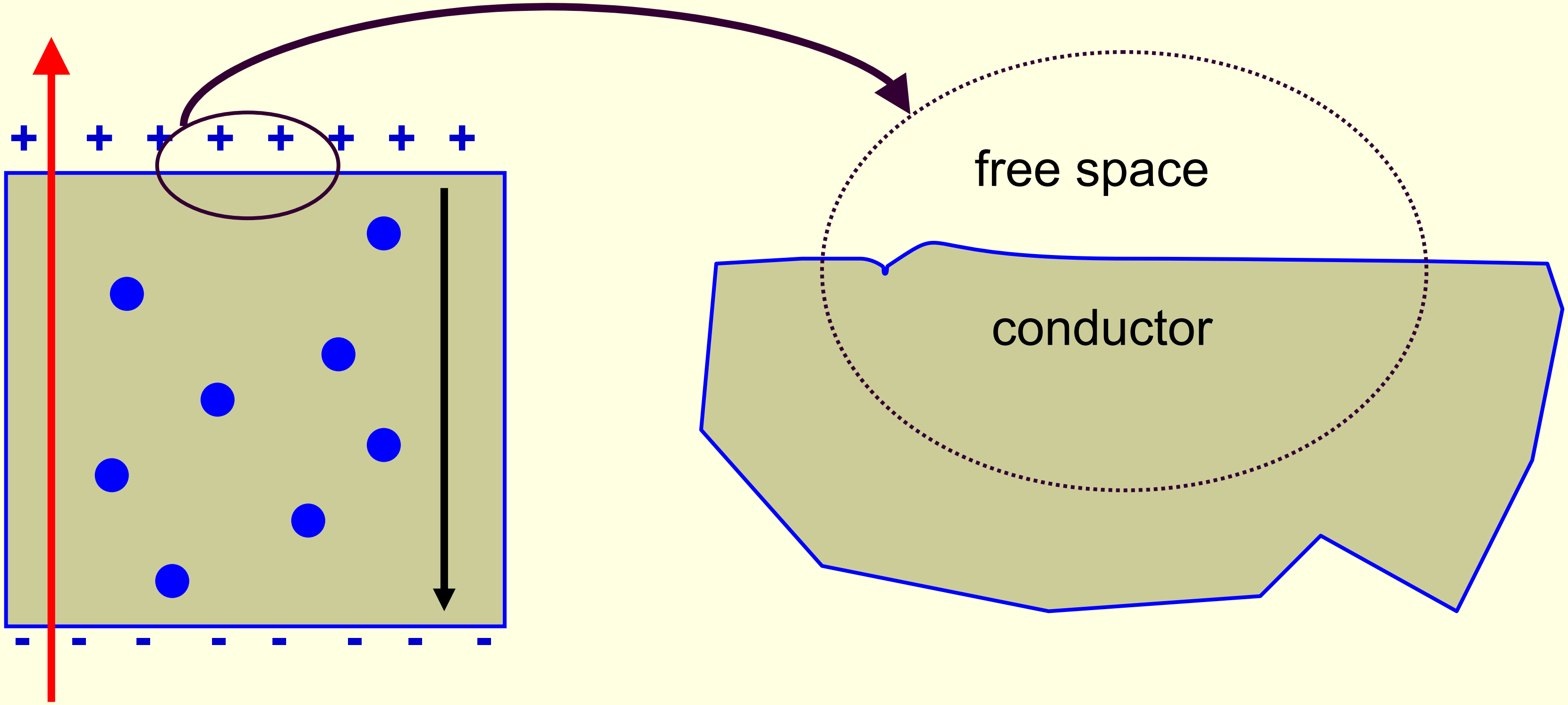
$$\Rightarrow \nabla V = -\vec{E} \Rightarrow$$ Equipotential body

CONDUCTOR PROPERTIES UNDER STATIC CONDITION

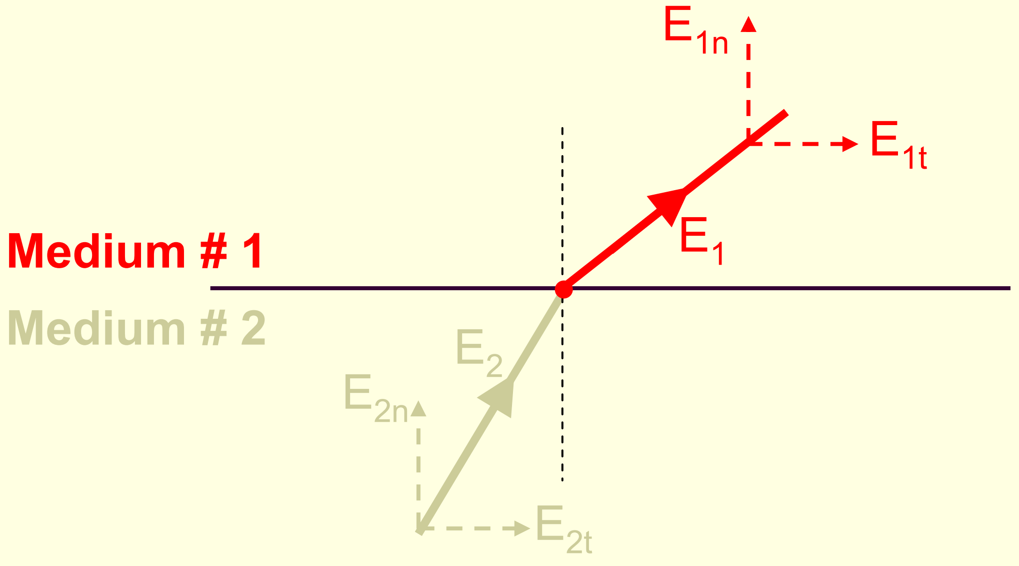
- The net $\rho_v = 0$ inside the conductor.
- The net **E** = 0 inside the conductor.
- The conductor is an equipotential body.
- Charge density ρ_s, if present, is found only on the surface.

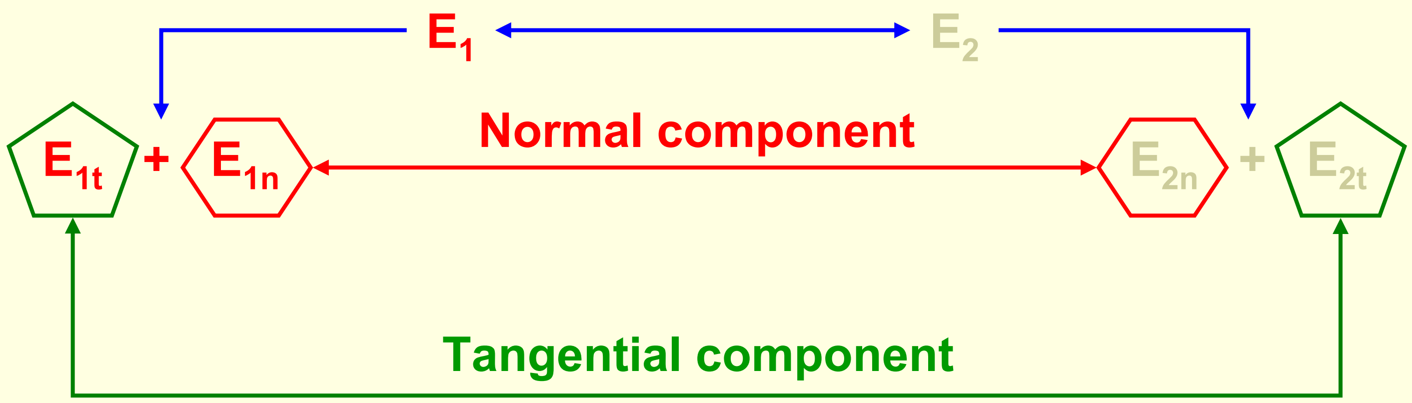
CONDUCTOR – FREE SPACE BOUNDARY CONDITIONS

Introduction



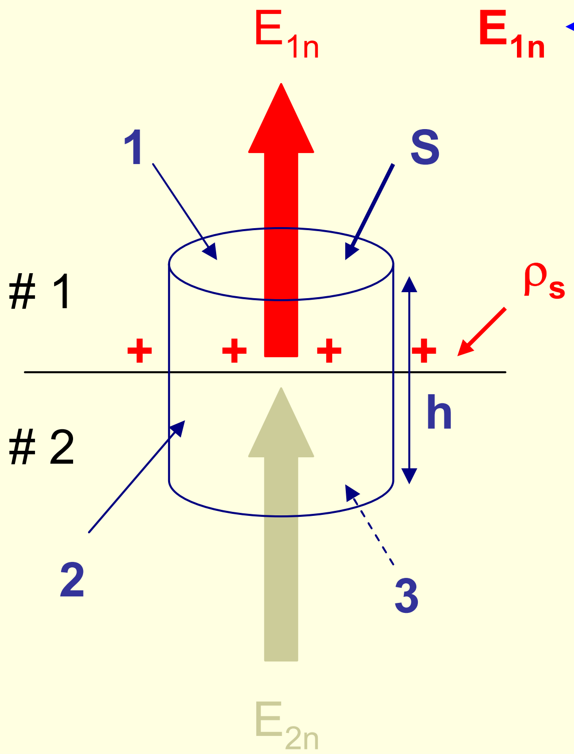
GRAPHICAL ILLUSTRATION





NORMAL COMPONENTS

E_{1n} ⟷ E_{2n}



Maxwell's equation (Gauss's law)

$$\oint_s \vec{D}\cdot d\vec{s} = Q_{en}$$

$$\int_s \rho_s ds$$

$$\int \vec{D}\cdot d\vec{s}_1 + \int \vec{D}\cdot d\vec{s}_2 + \int \vec{D}\cdot d\vec{s}_3$$

$D_{1n}\hat{z}$, $ds_1\hat{z}$, 0, $D_{2n}\hat{z}$, $ds_3(-\hat{z})$

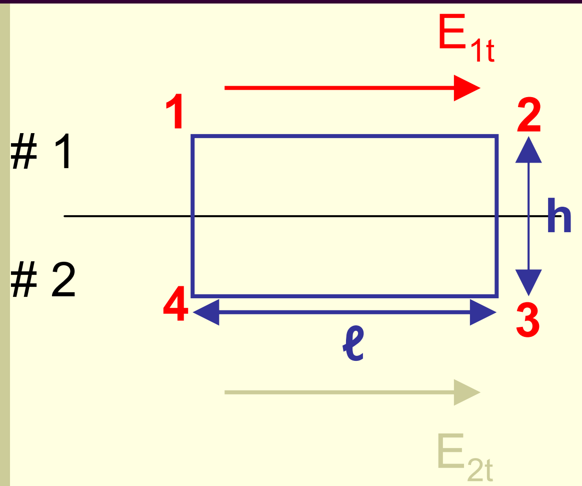
Assume h → 0

$$D_{1n}S - D_{2n}S = \rho_s S$$

Boundary condition for normal components

$$D_{1n} - D_{2n} = \rho_s$$

TANGENTIAL COMPONENTS



E_{1t} ↔ E_{2t}

Maxwell's equation (Conservation of energy)

$$\oint \vec{E}\cdot d\vec{\ell} = 0$$

$$\int_1^2 \vec{E}\cdot d\vec{\ell} + \int_2^3 \vec{E}\cdot d\vec{\ell} + \int_3^4 \vec{E}\cdot d\vec{\ell} + \int_4^1 \vec{E}\cdot d\vec{\ell}$$

0   0

Assume again h → 0

$$E_{1t}\ell - E_{2t}\ell = 0$$

Boundary condition for tangential components

$$E_{1t} = E_{2t}$$

CONDUCTOR – FREE SPACE BOUNDARY CONDITION

1 → Free space ε_0

2 → Conductor ε_0

Boundary condition for normal components

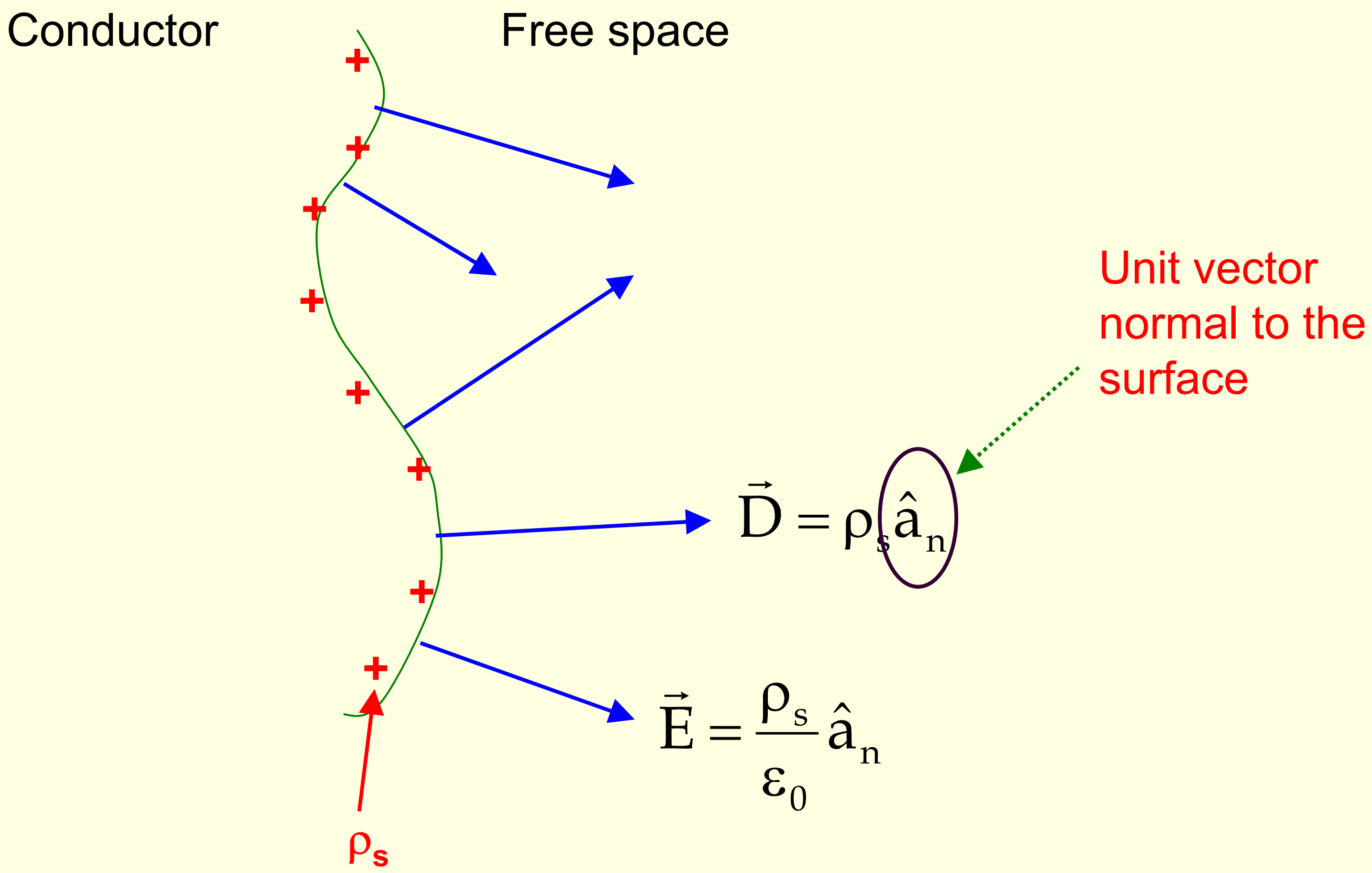
$$D_{1n} - D_{2n} = \rho_s \quad (D_{2n} = 0) \quad \Rightarrow \quad D_{1n} = \rho_s$$

Boundary conditions for conductor – free space/dielectric

Boundary condition for tangential components

$$E_{1t} = E_{2t} \quad (E_{2t} = 0) \quad \Rightarrow \quad E_{1t} = E_{2t} = 0$$

GRAPHICAL ILLUSTRATION

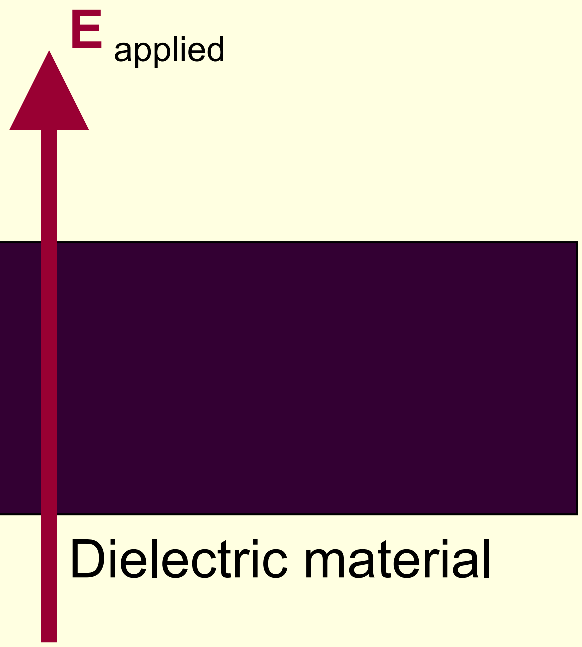


SUMMARIZED THE PRINCIPLES WHICH APPLY TO CONDUCTORS IN ELECTROSTATIC FIELDS

- The static electric field intensity inside a conductor is zero.
- The static electric field at the surface of a conductor is everywhere directed normal to that surface.
- The conductor surface is an equipotential surface.

DIELECTRIC MATERIALS

DIELECTRIC MATERIAL UNDER STATIC CONDITIONS

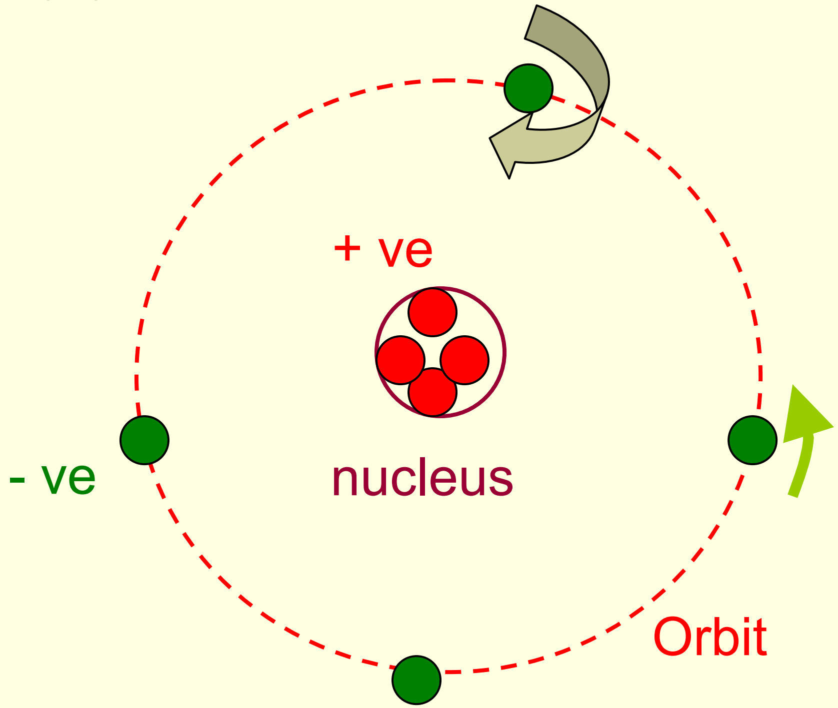


- APPROACH
  - Microscopic point of view
  - Macroscopic point of view

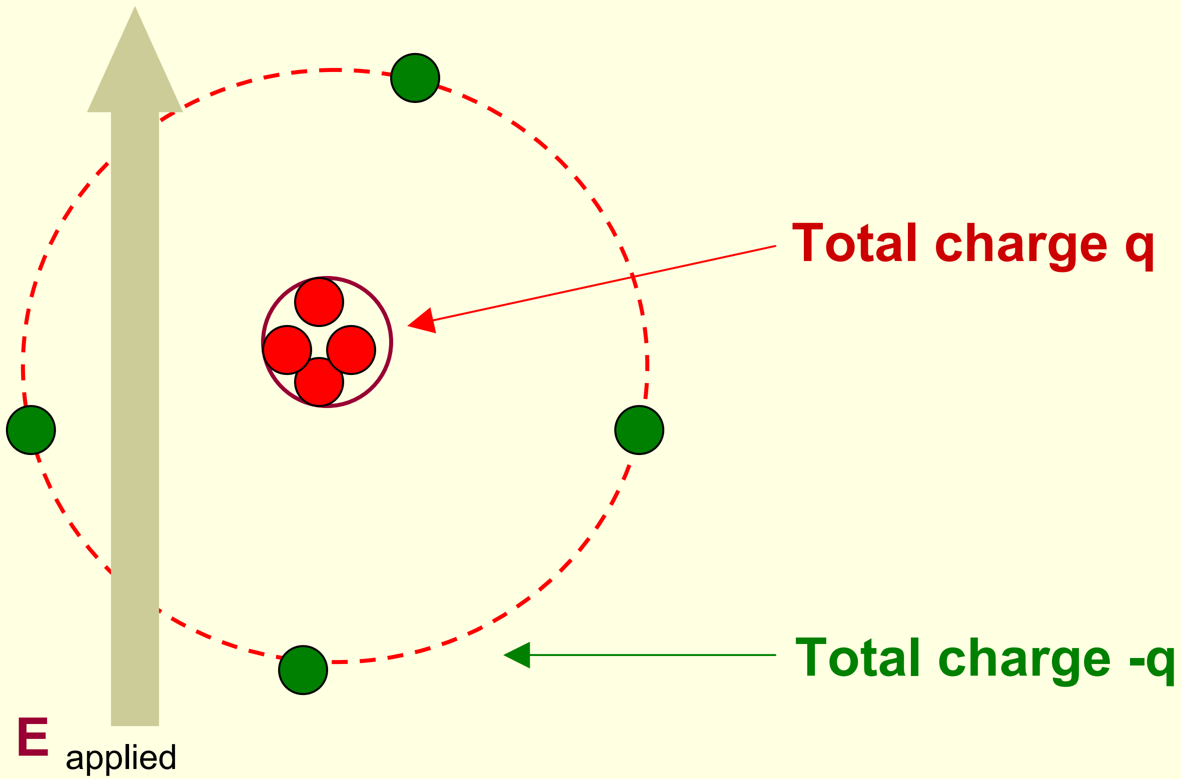
MICROSCOPIC POINT OF VIEW

Classical model of an atom

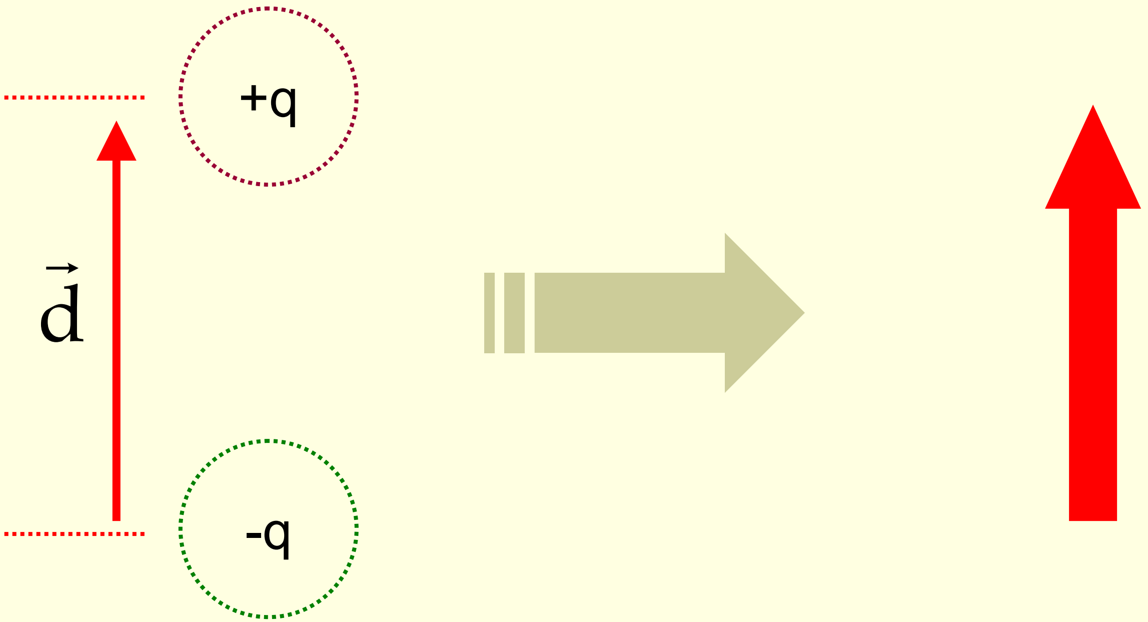
Assume $E_{applied} = 0$



AN ATOM UNDER STATIC CONDITION



MICROSCOPIC DIPOLE MOMENT

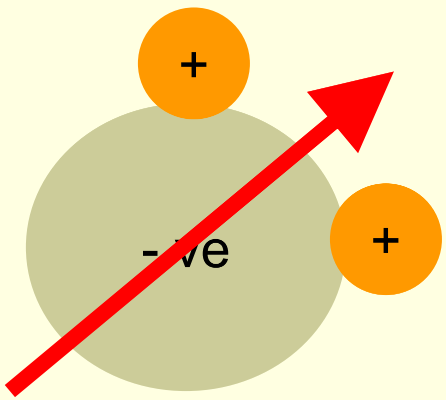


$$\vec{p} = q\vec{d}$$

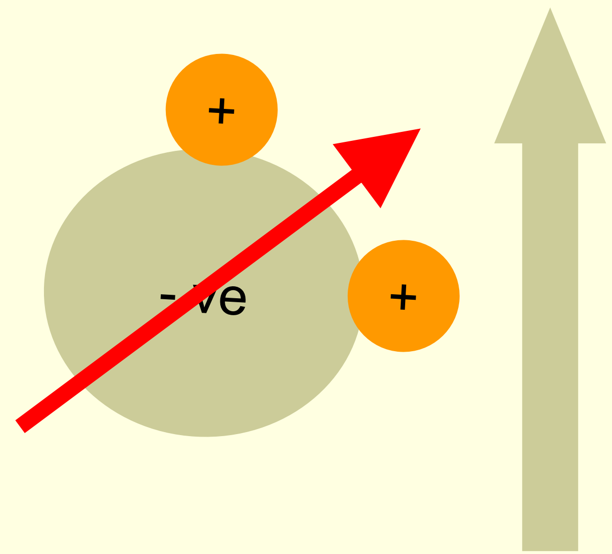
MACROSCOPIC POINT OF VIEW

Assume dielectric material is polar material

$H_2$0 molecule



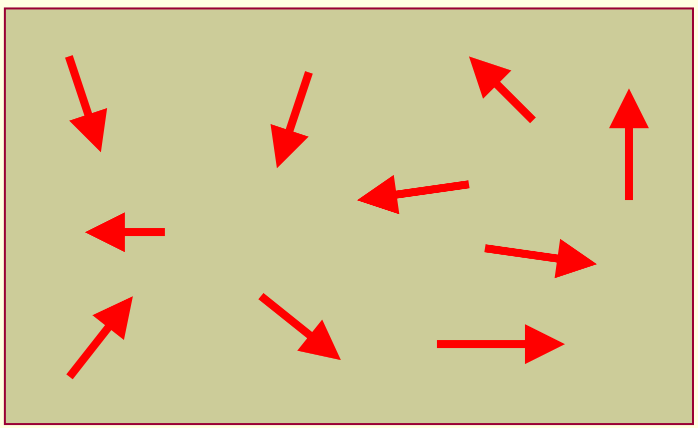
$E_{applied} = 0$



$E_{applied} \neq 0$

GROUP OF ATOMS

Assume number of atoms is 10



$E_{applied} = 0$

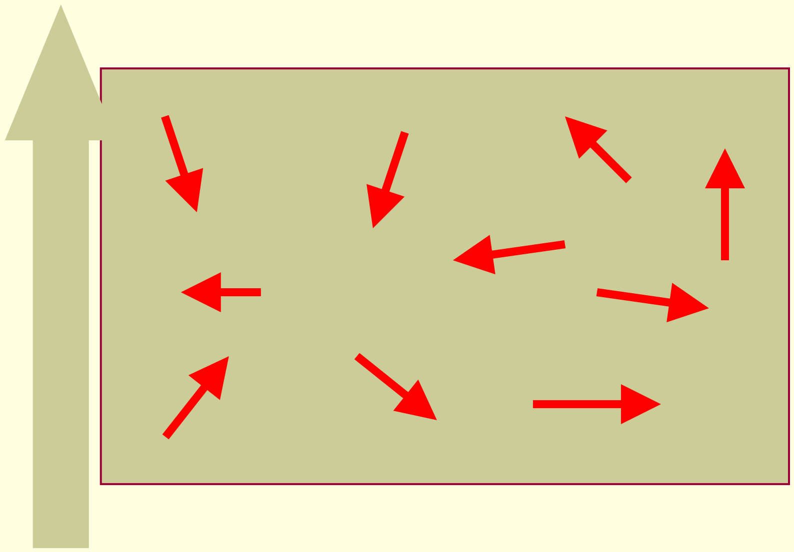
Electric polarization vector

$$\vec{P} = n\vec{p} = 0$$

# of atoms

Electric dipole moment

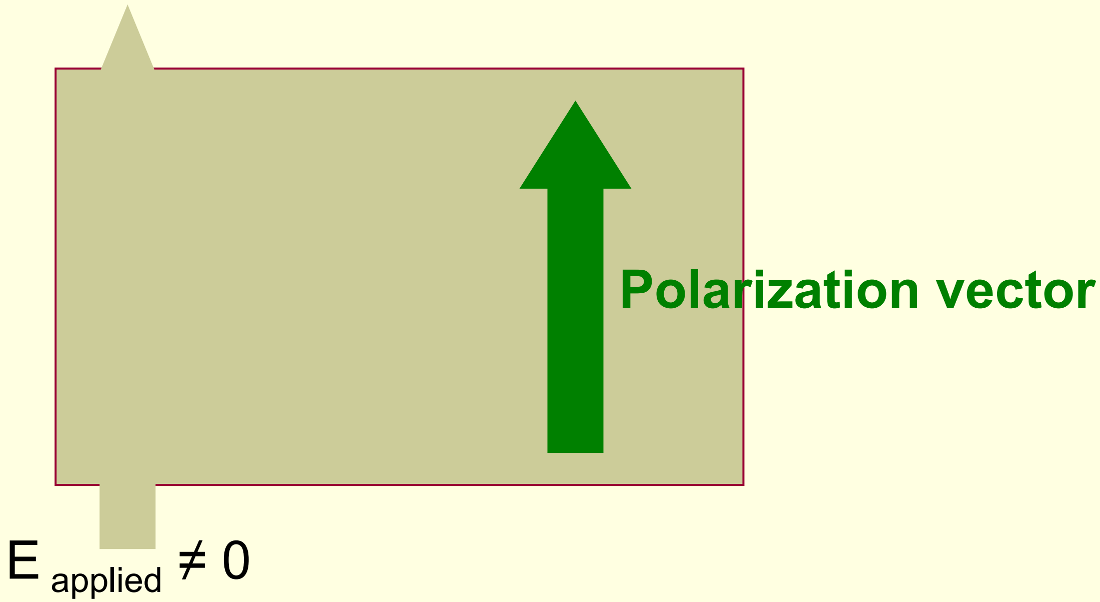
$E_{applied} \neq 0$



Polarized dielectric material.

$$\vec{P} = n\vec{p} \neq 0$$

REVIEW

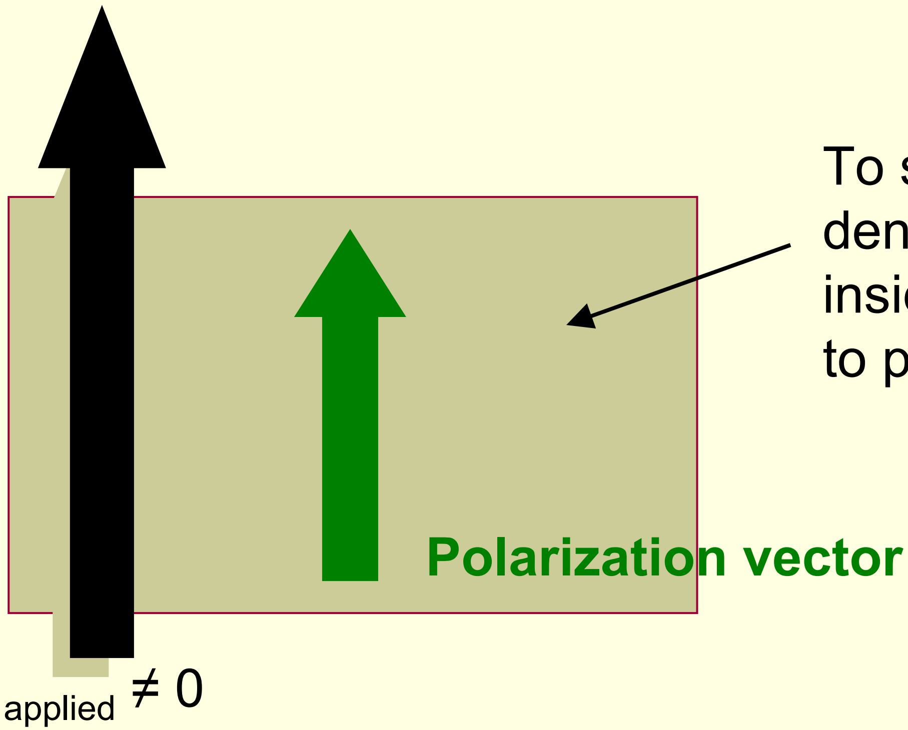


BOUND CHARGES

OBJECTIVES

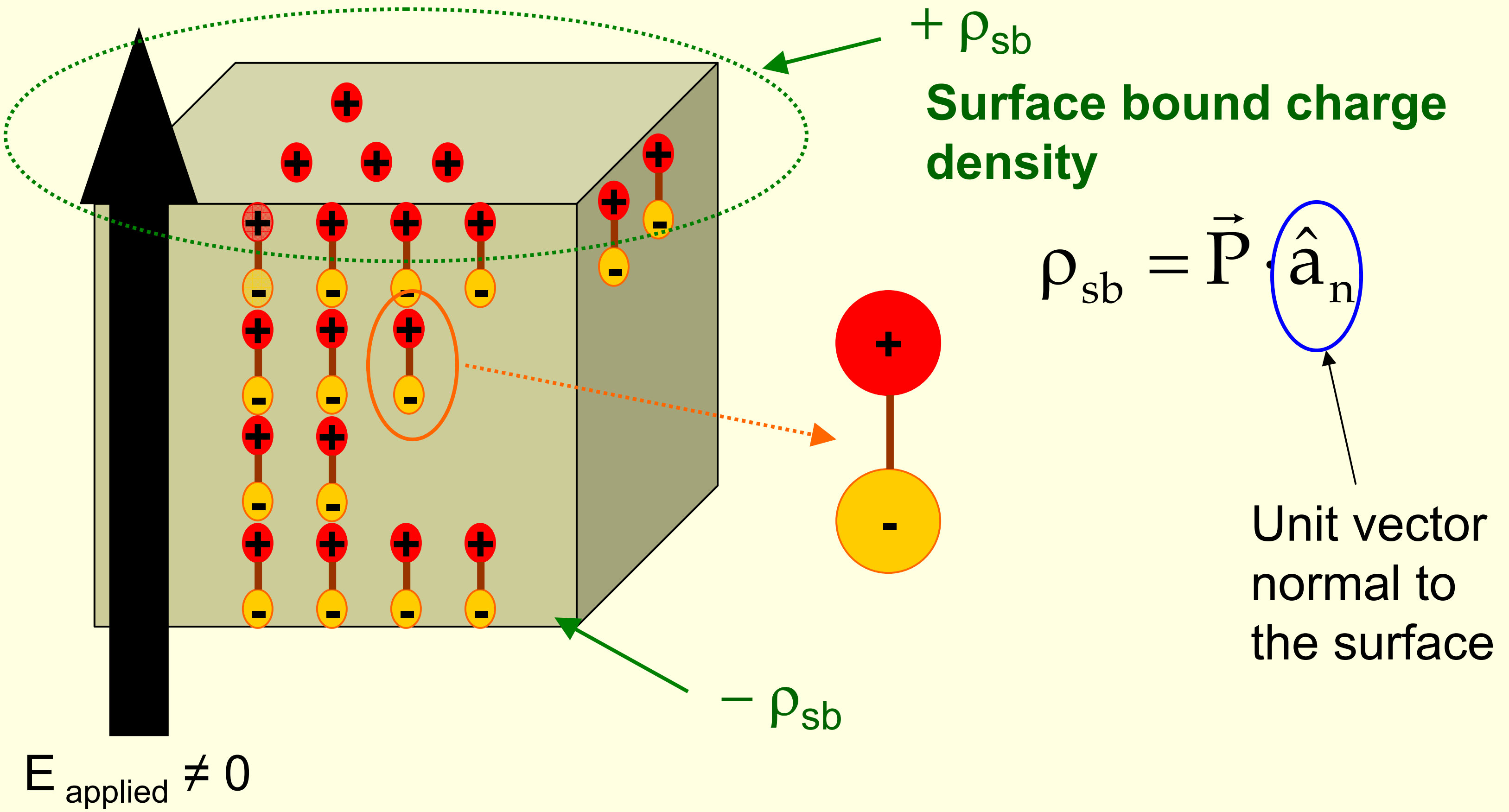
- To show ρ_{sb} and ρ_{vb} can be found within a dielectric due to polarization.
- To derive equations for ρ_{sb} and ρ_{vb}.

REVIEW



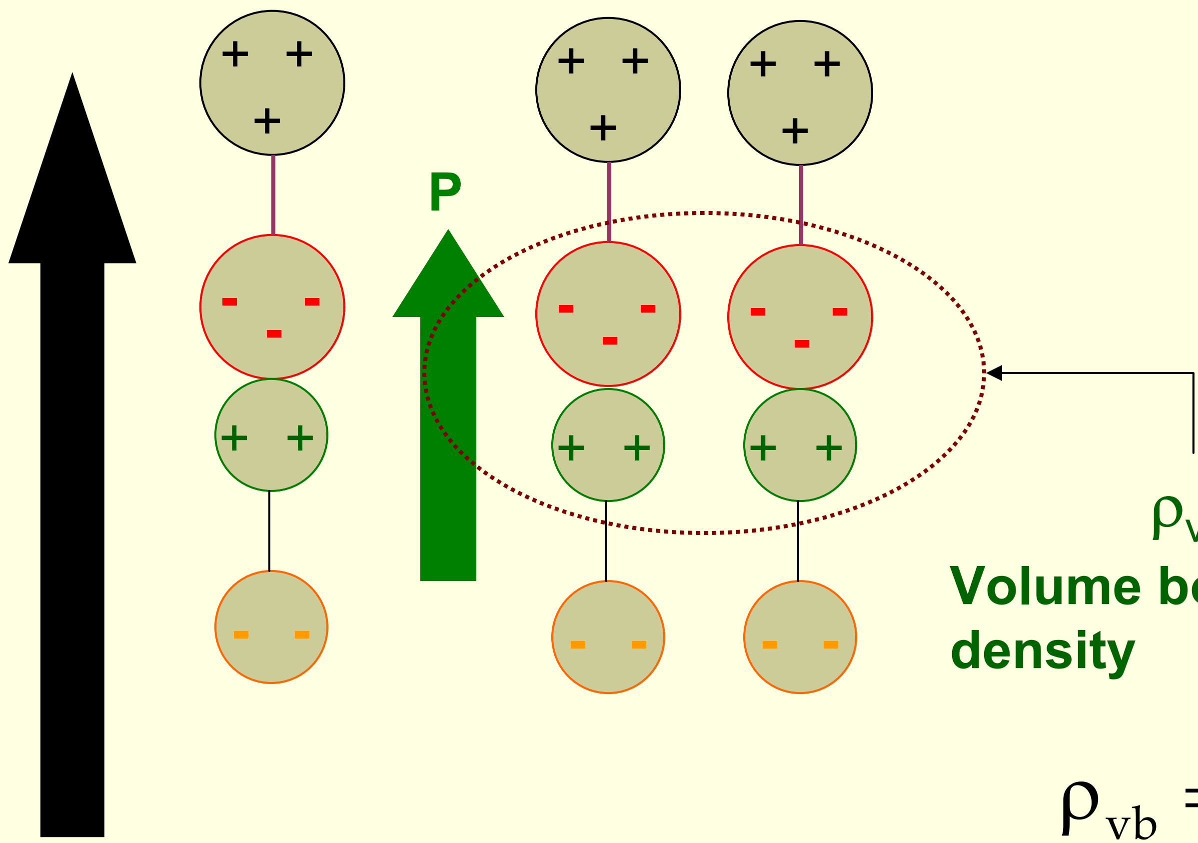
To show bound charges densities can be found inside a dielectric due to polarization.

GRAPHICAL ILLUSTRATION (ρ_{sb})



$$\rho_{sb} = \vec{P} \cdot \hat{a}_n$$

GRAPHICAL ILLUSTRATION (ρ_{vb})



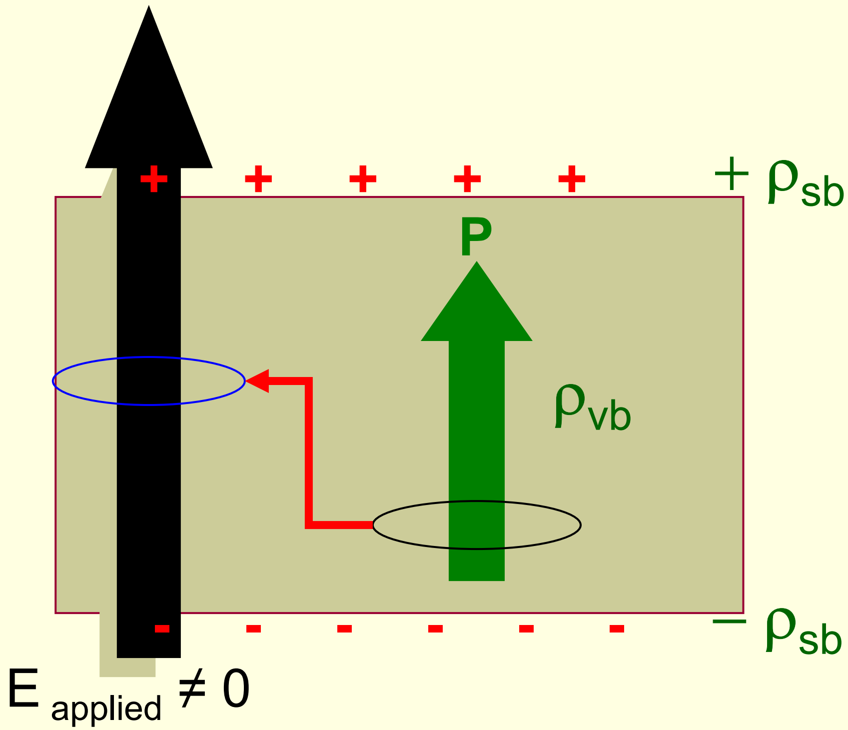
$E_{applied}$ non uniform

ρ_{vb}

Volume bound charge density

$$\rho_{vb} = -\nabla \cdot \vec{P}$$

EFFECT OF POLARIZATION ON FIELDS



MATHEMATICAL MODEL

Maxwell's equation (Point form of Gauss's law)

Free space

$$\nabla \cdot \vec{D} = \rho_v$$

$$\nabla \cdot \varepsilon_0 \vec{E} = \rho_v$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

General material

$$\nabla \cdot \varepsilon_0 \vec{E} = \rho_v + \rho_{vb}$$

$$\rho_{vb} = -\nabla \cdot \vec{P}$$

$$\nabla \cdot \left(\varepsilon_0 \vec{E} + \vec{P}\right) = \rho_v$$

To convert to more familiar form of equation

$$\vec{P} = \chi_e \varepsilon_0 \vec{E}$$

Electric susceptibility

$$\vec{D} = \varepsilon_0 (1 + \chi_e) \vec{E}$$

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} = \varepsilon \vec{E}$$

Permittivity

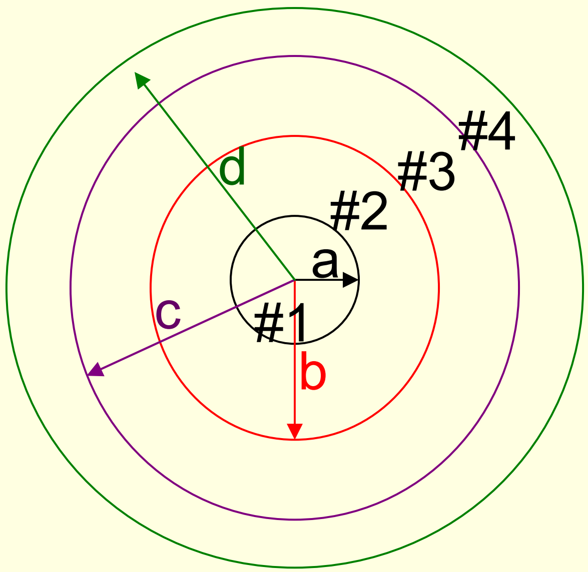
Relative permittivity ⟷ Dielectric constant

DIELECTRIC MATERIALS AT LOW FREQUENCY

Material	ε_r	**Material**	ε_r
Air	1.0006	Polyethylene	2.26
Bakelite	4.8	Polystyrene	2.5
Glass	6.0	Quartz	3.8
Lucite	3.2	Soil (dry)	3.0
Nylon	3.6	Teflon	2.1
Plexiglas	3.45	Water	80

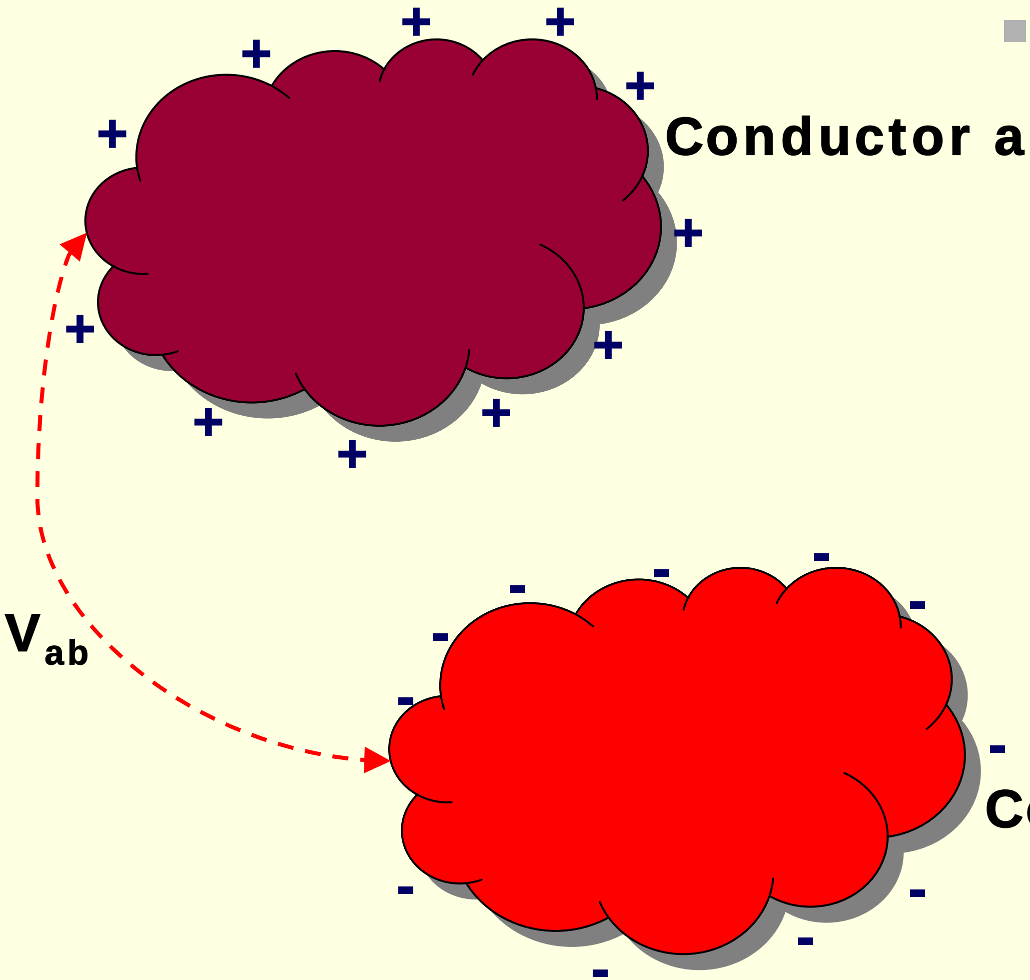
ASSIGNMENT

- An underground spherical configuration of concentric spherical dielectric shells, enclosed by a conductor shell, is shown below. Regions 1 and 3 are free space, region 2 is a dielectric whose relative permittivity is ε_r, and region 4 is a conductor. If charge Q is placed at the center, find: (i) **D**; (ii) **E**; (iii) **P**; (iv) ρ_s on the conductor surfaces; (v) ρ_{sb} on the dielectric surfaces; (vi) ρ_{vb} within the dielectric.



CAPACITANCE

GRAPHICAL ILLUSTRATION



- The capacitance of a two-conductor capacitor is defined as the ratio of the magnitude of charge on one of the conductors to the magnitude of the potential difference between the two conductors.

$$C = \frac{|Q|}{|V_{ab}|}$$

MATHEMATICAL MODEL

$$C = \frac{\left| \int_{s_a} \rho_s \, ds \right|}{\left| -\int_b^a \vec{E} \cdot d\vec{l} \right|}$$

$$C = \frac{\left| \int_{s_a} \varepsilon \vec{E} \cdot d\vec{s} \right|}{\left| -\int_b^a \vec{E} \cdot d\vec{l} \right|}$$

APPROACH

Choose a suitable coordinate system

↓

Let the two conducting plates carry charges +Q and –Q

↓

Determine E using Coulomb’s law or Gauss’s law

↓

Find V

↓

Obtain C from C = Q/V

SUMMARY

- The capacitance of a two-conductor body and resistance of the medium between them can be computed from knowledge of the electric field in that medium.